Hydropower Plants: Generating and Pumping Units
Solved Problems: Series 3

1 STUDY ABOUT A HYDRAULIC POWER PLANT

1.1 Basic calculation for a specific speed

Here, the procedure for choosing the type of turbine for a given power plant and the fundamentals of Pelton turbines are studied. In this exercise, the hydraulic power plant of interest has a rated discharge Q = 28 m^3 s^{-1} and an available head H = 350 m. For calculations, use the following values for gravity acceleration and water density:

$$g = 9.81 \text{ m s}^{-2}, \rho = 1'000 \text{ kg m}^{-3}$$

1) For the generating unit, it is planned to install a generator featuring 50 poles. Deduce the rotational frequency n of the turbine. For the grid frequency f_{grid}, use the value of f_{grid} = 60 Hz.

$$n = \frac{2 f_{grid}}{z_p} = 2.40 \text{ (Hz)}$$

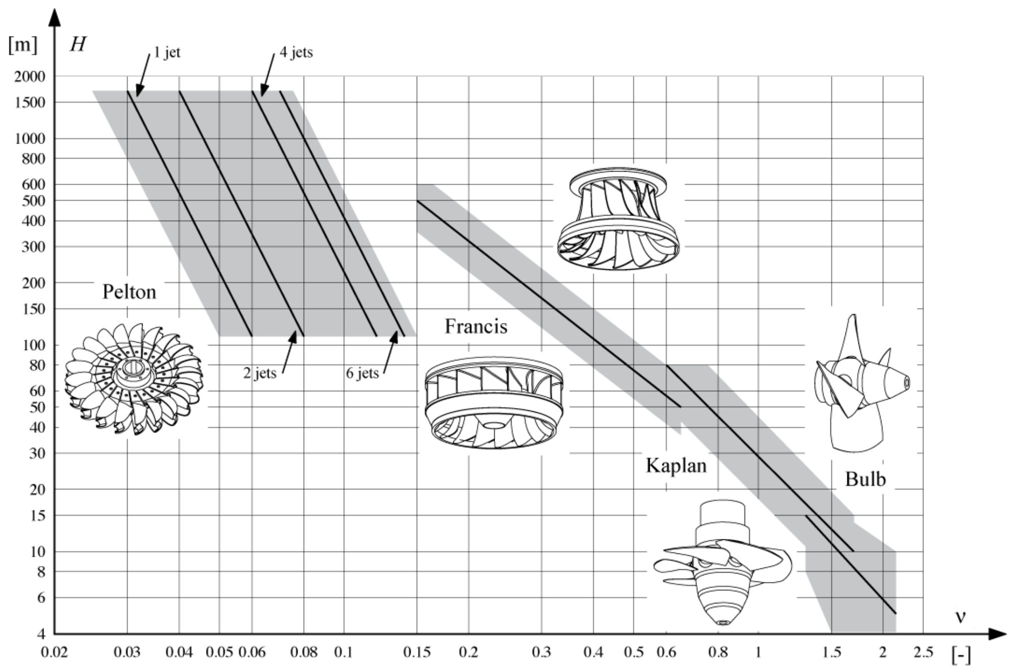

Figure 1: Turbine type depending on specific speed ν and head H

2) Calculate the specific speed ν and, using Figure 1, select a suitable turbine type for the power plant.

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Using the definition of the specific speed and the relation E = gH, the specific speed is, knowing that $Q = 28\ m^3\ s^{-1}$, H = 350 m and n = 2.4 Hz:

$$\nu = 2^{\frac{1}{4}} \pi^{\frac{1}{2}} n \frac{Q^{\frac{1}{2}}}{E^{\frac{3}{4}}} = 0.0597$$

Using the rotational frequency $\omega = 2\pi n$, the specific speed can also be expressed as:

$$\nu = \frac{\omega Q^{\frac{1}{2}}}{\pi^{\frac{1}{2}} (2E)^{\frac{3}{4}}} = 0.0597$$

Therefore, according to Figure 1, a two jets Pelton turbine is the most suitable runner for a power plant with $n \cong 0.06$ and H = 350 m

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1.2 Study about a Pelton turbine

Pelton turbines are mostly used for high head hydropower plants. Their geometry is based on water splitting buckets (see Figure 2). Answer the following questions using the values provided in Section 1.1.

3) In which head range are Pelton turbines used?

In the range of 300 to 2'000 m.

4) What is the difference between horizontal and vertical axis Pelton turbines? For a large-sized Pelton runner, explain which one is the most appropriate and why.

In the case of a horizontal shaft, the force acting on the bearing is not circumferentially equal due to the action of gravity. That's why it is better to use a vertical shaft in the case of a large Pelton turbine.

5) Explain the difference between Pelton and Francis turbines using the following keywords: *impulse turbine, reaction turbine, velocity, pressure*

Pelton turbines are classified as impulse turbines, meaning that they only use the discharge velocity to produce energy. On the other hand, reaction turbines such as Francis, Kaplan and bulb turbines use both velocity and pressure to produce energy. Therefore, in the case of reaction turbines, the pressure gradually decreases along the streamline, i.e. along the blades of the turbine. By contrast, the pressure is set at atmosphere pressure in the case of impulse turbines.

For Pelton turbines, an output power *P* can be calculated by the following equation based on the velocity triangle in Figure 2, when the bucket and mechanical losses are negligible:

$$P = \rho Q \left\{ \frac{\left(\vec{W}_1 + \vec{U}_1\right)^2}{2} - \frac{\left(\vec{W}_{\bar{1}} + \vec{U}_{\bar{1}}\right)^2}{2} \right\} = \rho Q \left(\vec{W}_1 \cdot \vec{U}_1 - \vec{W}_{\bar{1}} \cdot \vec{U}_{\bar{1}} \right) \quad (1)$$

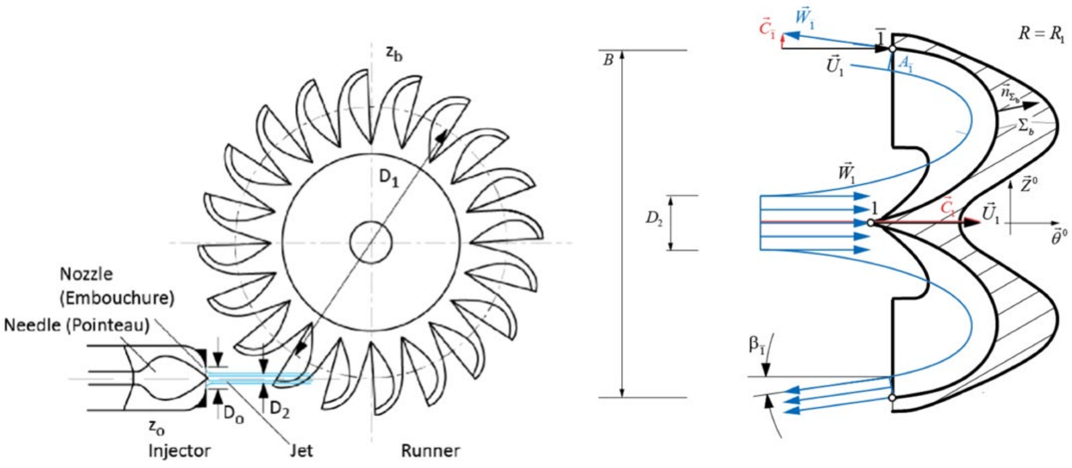


Figure 2: Pelton turbine bucket and velocity triangle ($\vec{C}$ the absolute discharge velocity, $\vec{U}$ the bucket tangential velocity, $\vec{W}$ the relative discharge velocity)

6) Deduce an expression equivalent to $\vec{W}_1 \cdot \vec{U}_1 - \vec{W}_{\bar{1}} \cdot \vec{U}_{\bar{1}}$ and express it as a function of C_1, U_1 and $\beta_{\bar{1}}$ based on the vectorial relationship illustrated in Figure 2. Here, the amplitude of the relative velocity $\left|\vec{W}_{\bar{1}}\right|$ can be assumed the same as $\left|\vec{W}_1\right|$.

Considering the vectorial relationship $W_1 = C_1 - U_1$:

$$\vec{W}_1 \cdot \vec{U}_1 - \vec{W}_{\bar{1}} \cdot \vec{U}_{\bar{1}} = W_1 U_1 + W_{\bar{1}} U_{\bar{1}} \cos\beta_{\bar{1}} = W_1 U_1 + W_1 U_1 \cos\beta_{\bar{1}} = \left(1+\cos\beta_{\bar{1}}\right)\left(C_1 - U_1\right)U_1$$

7) The jet velocity is $C_1 = \varphi_s \sqrt{2gH}$ with φ_s being a velocity coefficient, taking nozzle and volute (if present) losses into account. Assuming $\varphi_s = 0.97$, calculate the nozzle efficiency. Hint: Nozzle efficiency may be formulated as $\eta_s = \dfrac{0.5C_1^2}{gH}$.

Using the mentioned equation for nozzle efficiency:

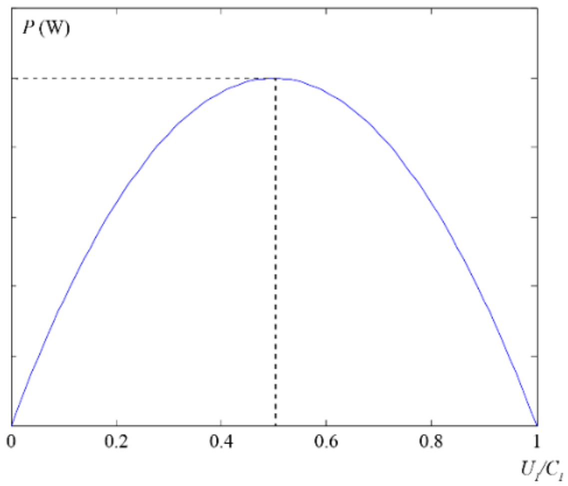
$$\eta_s = \frac{0.5C_1^{\,2}}{gH} = \frac{0.5\varphi_s^{\,2}\,2gH}{gH} = \varphi_s^{\,2} = 0.94$$

8) From Eq. (1) and the expression you computed in question 6), deduce the output power P as a function of the velocity ratio $\dfrac{U_1}{C_1}$. Then, plot the curve of the output power P as a function of the velocity ratio, and check the optimal bucket velocity $U_{1,opt}$. What do the points $\dfrac{U_1}{C_1} = 0$ and $\dfrac{U_1}{C_1} = 1$ represent?

According to Eq. (1) and Q. 6), the power can be expressed as;

$$P = \rho Q\left(\vec{W}_1 \cdot \vec{U}_1 - \vec{W}_{\bar{1}} \cdot \vec{U}_{\bar{1}}\right) = \rho Q\left(1+\cos\beta_{\bar{1}}\right)\left(C_1 - U_1\right)U_1 = \rho Q\left(1+\cos\beta_{\bar{1}}\right)\left(1-\frac{U_1}{C_1}\right)\frac{U_1}{C_1}C_1^{\,2}$$

The curve of the output power as a function of the velocity ratio is shown below:



The maximum power is generated at $\frac{U_1}{C_1} = 0.5 \Rightarrow U_{1,opt} = \frac{1}{2}C_1$.

The point $\frac{U_1}{C_1} = 0$ *corresponds to a startup where the runner is blocked, the torque being maximum and the power zero.*

The point $\frac{U_1}{C_1} = 1$ *corresponds to runaway, where the tangential velocity is equal to the jet speed. In this operating mode, the water jet hitting the bucket is making the runner spin without producing electricity as it is not connected to the grid. The power and torque become zero.*

9) Calculate the output power P for the designed power plant, at the velocity ratio $\frac{U_1}{C_1} = 0.6$ with the jet diameter $D_2 = 0.4709$ m and the outlet flow angle $\beta_{\bar{1}} = 5°$. Furthermore, calculate the maximum output power P_{max}.

Using the relation which links discharge and absolute velocity Q = CA, and the equation deduced in Q. 8) for P, we first compute C_1*:*

$$C_1 = \frac{\frac{Q}{2}}{\frac{\pi D_2^2}{4}} \cong \frac{14}{0.17423} \cong 80.39 \text{ m s}^{-1}$$

$$P = 1000 \times 28 \times (1 + \cos 5°) \times (1 - 0.6) \times 0.6 \times 80.39^2 \approx 86.69 \times 10^6 \text{ (W)}$$

The maximum power can be calculated:

$$P_{max} = 1000 \times 28 \times (1 + \cos 5^o) \times \frac{C_1^2}{4} = 90.30 \times 10^6 \text{ (W)}$$

Note: you can also calculate the jet speed C_1 *at the rated discharge by using the equation in the question (13), as in;*

$$C_1 = \varphi_s \sqrt{2gH} = 0.97 \times \sqrt{2 \times 9.81 \times 350} \cong 80.38 \,\text{m s}^{-1}$$

10) What is the mechanical risk for the generator when connecting the turbine to the grid at runaway?

Over-speed, runner unbalance, vibrations, and risk of contact or sticking between rotor and stator.

11) Express the optimum diameter of the Pelton runner (i.e. consistent with the maximum output power condition) as a function of the available head H and angular rotational frequency ω.

The maximum power is generated at $\frac{U_1}{C_1} = 0.5$ *, thus:*

$$U_{1,opt} = \frac{C_1}{2} \text{ and } U_1 = \omega R_1 = \omega \frac{D_1}{2} \Rightarrow D_{1,opt} = \frac{2U_{1,opt}}{\omega} = \frac{C_1}{\omega}$$

And, using $C_1 = \varphi_s \sqrt{2gH}$ *:*

$$D_{1,opt} = \frac{\varphi_s \sqrt{2gH}}{\omega}$$

2 APPLICATION OF EULER EQUATION TO PELTON TURBINE

The Euler equation can be applied to the calculation of the transformed specific energy E_t for Pelton turbines. For these turbines, the transformed specific energy E_t can be calculated in the following way:

$$E_t = \vec{C}_1 \cdot \vec{U}_1 - \vec{C}_{\bar{1}} \cdot \vec{U}_{\bar{1}}$$

In Figure 3, a sketch of the flow in a Pelton bucket is shown, as well as the corresponding velocity triangles (Q: discharge, C: absolute jet velocity, U: absolute runner rotational velocity, W: relative velocity).

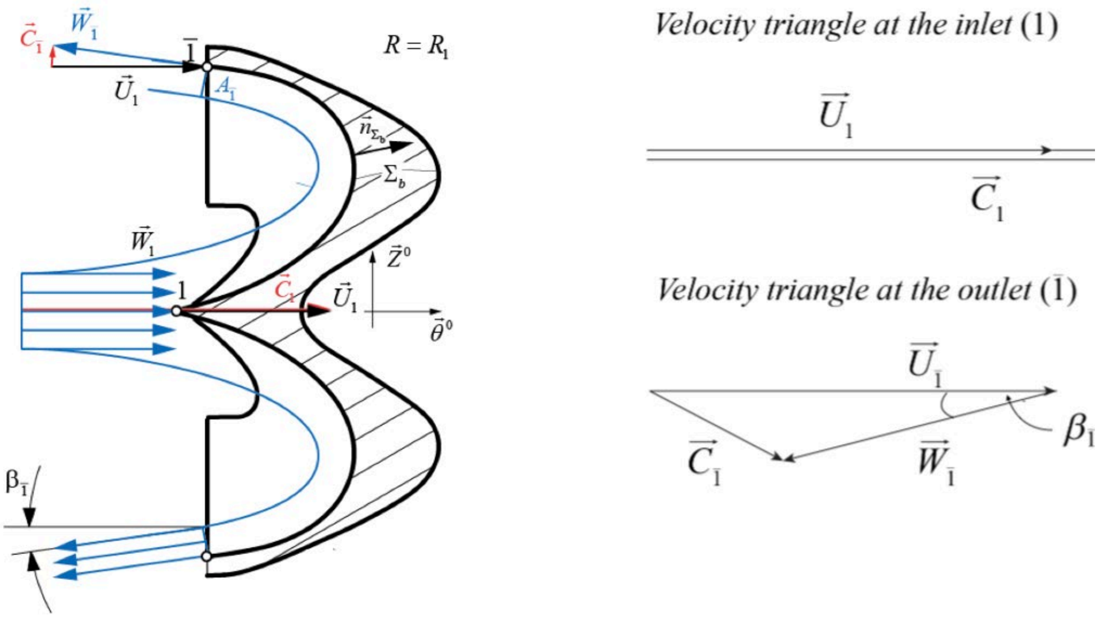


Figure 3: Sketch of the flow in a Pelton bucket, with the corresponding velocity triangles at the inlet and the outlet.

12) Using the Euler equation and the velocity triangles at the inlet and the outlet, express the transformed specific energy E_t as a function of the relative velocity $W_{\bar{1}}$, the absolute jet velocity C_1, the rotational velocity of the runner U and the angle $\beta_{\bar{1}} = \beta$.

Starting from the Euler equation for a Pelton turbine, and using $\overrightarrow{C_{\bar{1}}} = \overrightarrow{U_{\bar{1}}} + \overrightarrow{W_{\bar{1}}}$ as well as the outlet velocity triangle, we get:

$$E_t = C_1 U_1 - \left(U_{\bar{1}} - W_{\bar{1}} \cos \beta_{\bar{1}}\right) U_{\bar{1}}$$

And knwoing that the circumferential speed U is the same at the inlet and outlet:

$$E_t = C_1 U - \left(U - W_{\bar{1}} \cos \beta\right) U$$

13) Assuming that $\left|\overrightarrow{W_1}\right| = \left|\overrightarrow{W_{\bar{1}}}\right|$, give the transformed specific energy E_t as a function of C_1, U and β.

Using the previous answer and applying $W_{\bar{1}} = W_1$ as well as the relationship coming from the velocity triangle at the inlet ($W_1 = C_1 - U_1$):

$$E_t = C_1 U - \left(U - \left(C_1 - U\right) \cos \beta\right) U = U\left(C_1 - U\right)\left(1 + \cos \beta\right)$$

14) Express the transferred power P_t as a function of ρ, Q, C_1, U and β. You can consider $Q_t = Q$.

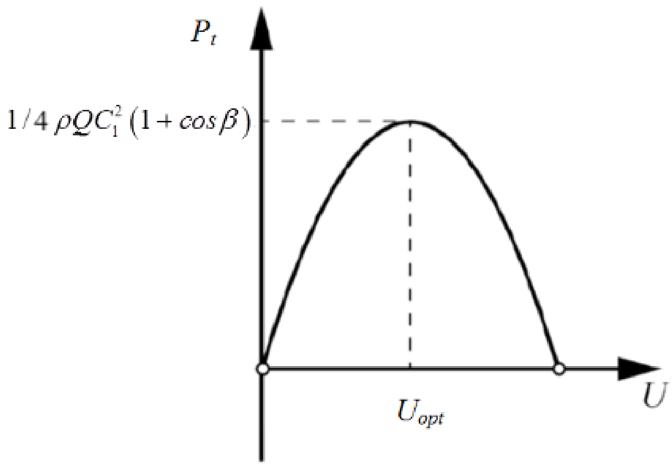
$$P_t = \rho Q E_t = \rho Q U\left(C_1 - U\right)\left(1 + \cos \beta\right)$$

15) If your calculations are correct, you should end up with the same expression of the power as in Question 8). However, here we considered the transferred power P_t, whereas in Q.8) we referred to the output power *P*. Which assumption made in Q.5) causes these two power expressions to be the same?
On the other hand, which assumption from Q.5) doesn't have an impact on the difference that actually exists between the transferred power P_t and the output power *P*?

In Q.5) we neglected the bucket losses and mechanical losses. For the transferred and the output power to be equal, only neglecting the mechanical losses (disk losses and bearing losses, i.e. the losses generated by rotational frictions) has an influence.

On the other hand, neglecting the bucket losses (i.e. friction of the water on the bucket surface) leads to an underestimation of the specific hydraulic energy losses. This losses term is responsible for the decrease in transferred power, which is lower than the hydraulic power.

16) For a given discharge Q and absolute jet velocity C_1, sketch the evolution of P_t as a function of U.



Evolution of the transferred power P_t with respect to U

17) Express the maximum transformed power $P_{t_{max}}$ and the optimal runner velocity U_{opt} at the maximum power as a function of ρ, Q, C_1 and β.

$$U_{opt} = \frac{1}{2} C_1$$

$$P_{t_{max}} = \frac{1}{4} \rho Q C_1^2 (1 + \cos\beta)$$